JESUS COLLEGE

ADMISSIONS TEST (MATHEMATICS)

SPECIMEN TEST

There are seven questions below which are on various areas of mathematics. They are of varying levels of difficulty: some may be easy and others could be hard.

You are not expected to answer all of them, or necessarily to give complete answers to questions, though complete answers are preferred to fragments.

You should begin each question on a new sheet of paper, and your work on this test will be used as a basis for discussion in the interview.

(1) Calculate $\int_0^\pi (x \sin x)^2 \, dx$.

(2) By sketching appropriate graphs, find all solutions to the equation $x - 1 = (e - 1) \ln x$. Hence sketch the graph of $f(x) = e^x - x^e$. (Here $\ln x$ denotes the logarithm to base e.)

(3) A cylindrical spaceship of mass M and cross-sectional area A is coasting at constant velocity when it suddenly encounters a dust cloud. The captain is dismayed to find that the dust sticks to the spaceship. If the density of dust is ρ, how far does the ship travel before its velocity is reduced by half?

(4) Twenty balls are placed in an urn. Five are red, five green, five yellow and five blue. Three balls are drawn from the urn at random without replacement. Write down expressions for the probabilities of the following events. (You need not calculate their numerical values.)

(i) Exactly one of the balls drawn is red.

(ii) The three balls drawn have different colours.

(iii) The number of blue balls drawn is strictly greater than the number of yellow balls drawn

(5) Of the numbers 1, 2, 3, ..., 6000, how many are *not* multiples of 2, 3 or 5?

(6) In a tennis tournament there are $2n$ participants. In the first round of the tournament, each player plays exactly once, so there are n games. Show that the pairings for the first round can be arranged in exactly $\dfrac{(2n-1)!}{2^{n-1}(n-1)!}$ ways.

(7) By considering the graph of the function $f(x) = x^{-s}$ or otherwise, show that for $s > 1$,

$$\frac{1}{s-1} \leqslant 1 + 2^{-s} + 3^{-s} + \cdots \leqslant \frac{s}{s-1}$$

With thanks to Trinity College for these specimen questions.